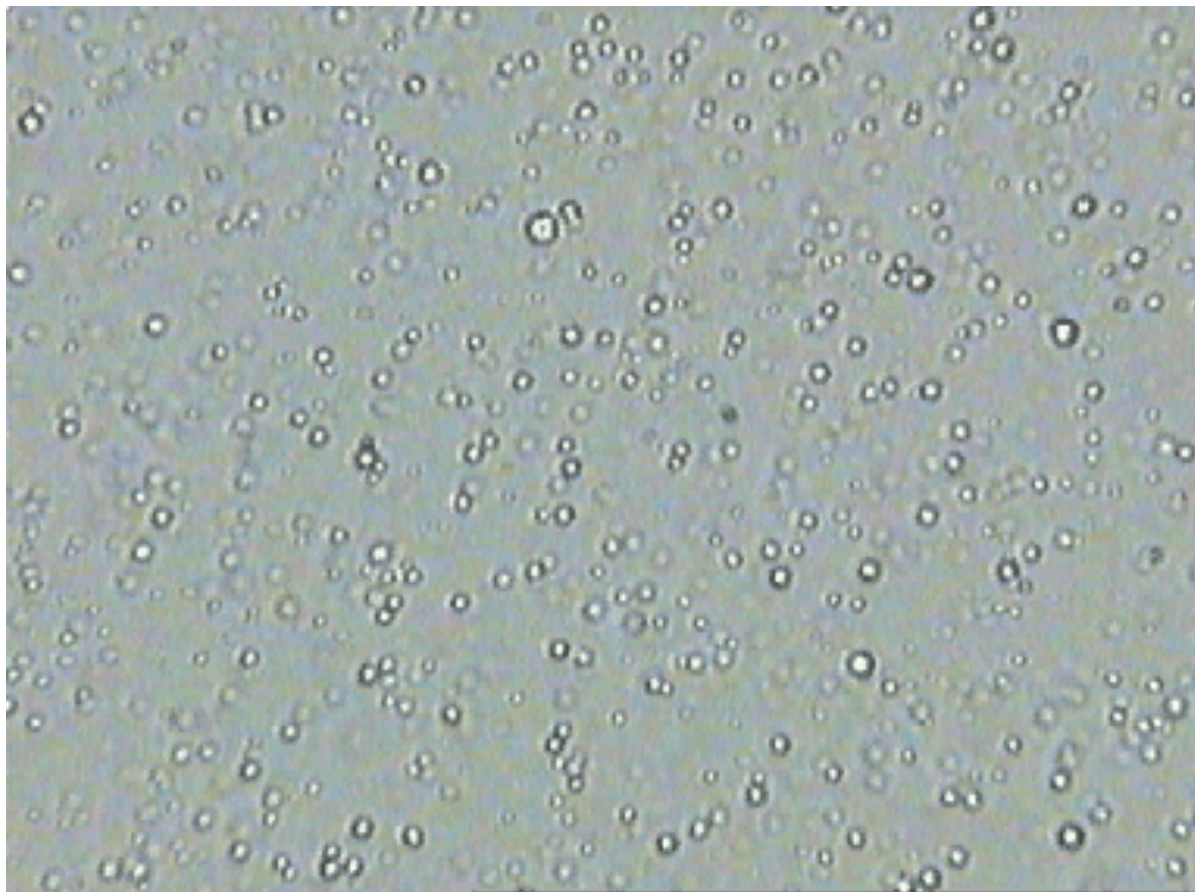# Course summary

18.354

Kepler's problem
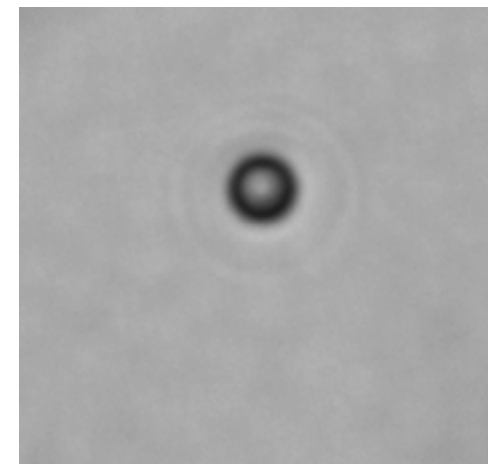
$$\mathbf{L} = \mathbf{r} \times m\frac{d\mathbf{r}}{dt}$$

Random walks & diffusion

David Walker

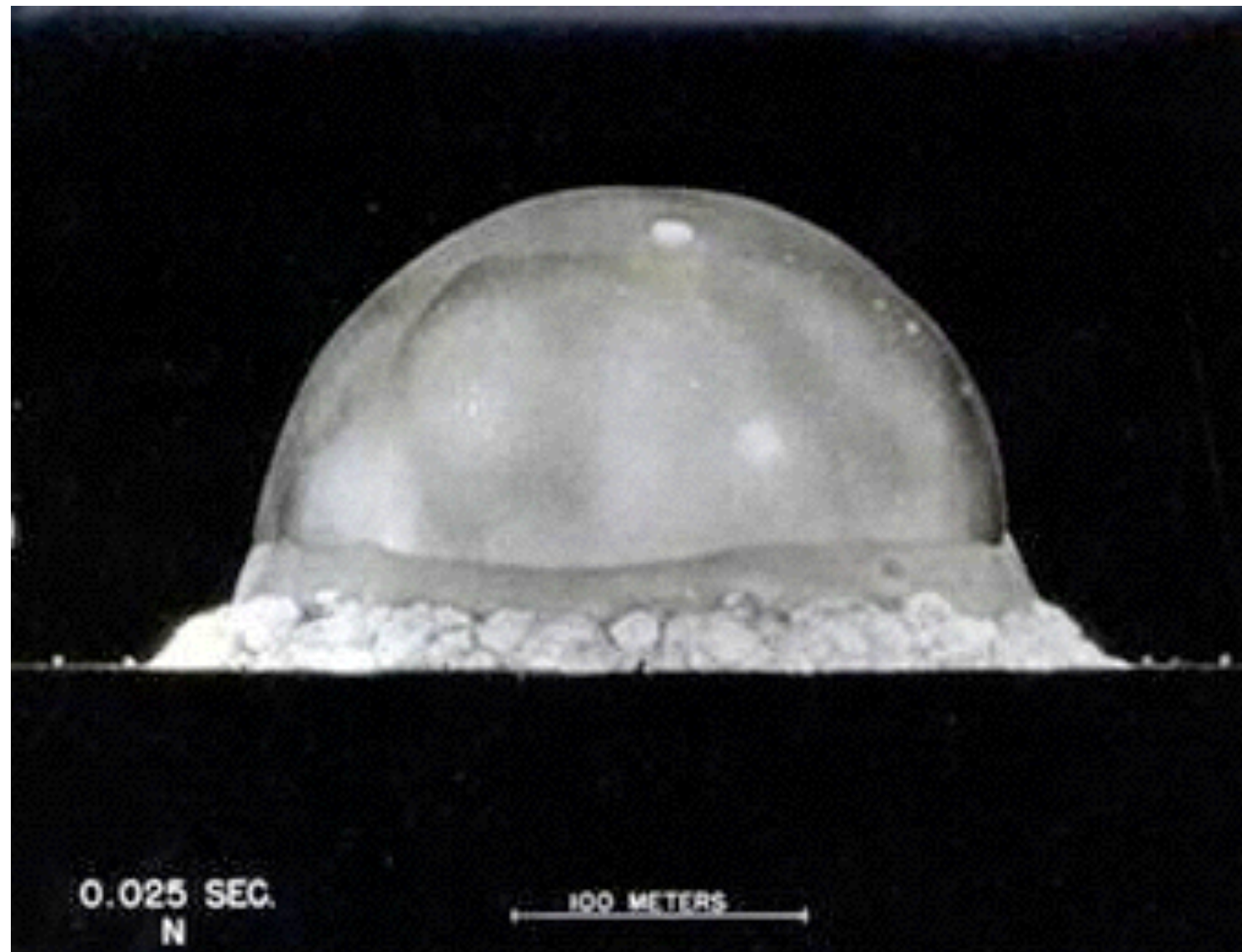$$\frac{\partial n}{\partial t} = -\frac{\partial J_x}{\partial x} = D\frac{\partial^2 n}{\partial x^2},$$
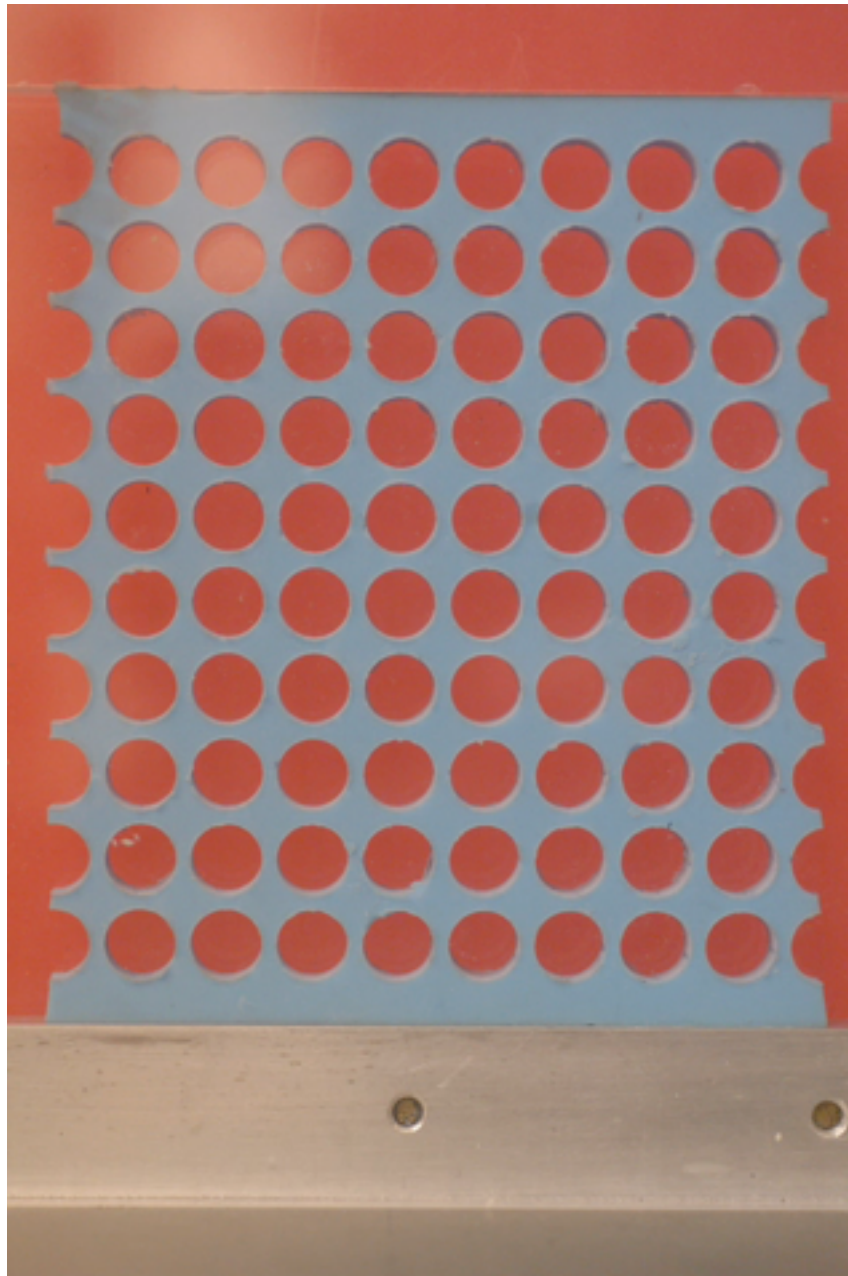

Mark Haw

Hydrodynamics

$$\int_V \frac{\partial \rho}{\partial t} dV = -\int_S \rho \mathbf{u} \cdot \mathbf{n} dS = -\int_V \nabla \cdot (\rho \mathbf{u}) dV.$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0.$$

$$\int_{V(t)} \rho \frac{D\mathbf{u}}{Dt} dV = \int_{V(t)} (-\nabla p + \rho \mathbf{g}) dV$$

$$\frac{D\mathbf{u}}{Dt} = \frac{-\nabla p}{\rho} + \mathbf{g}.$$

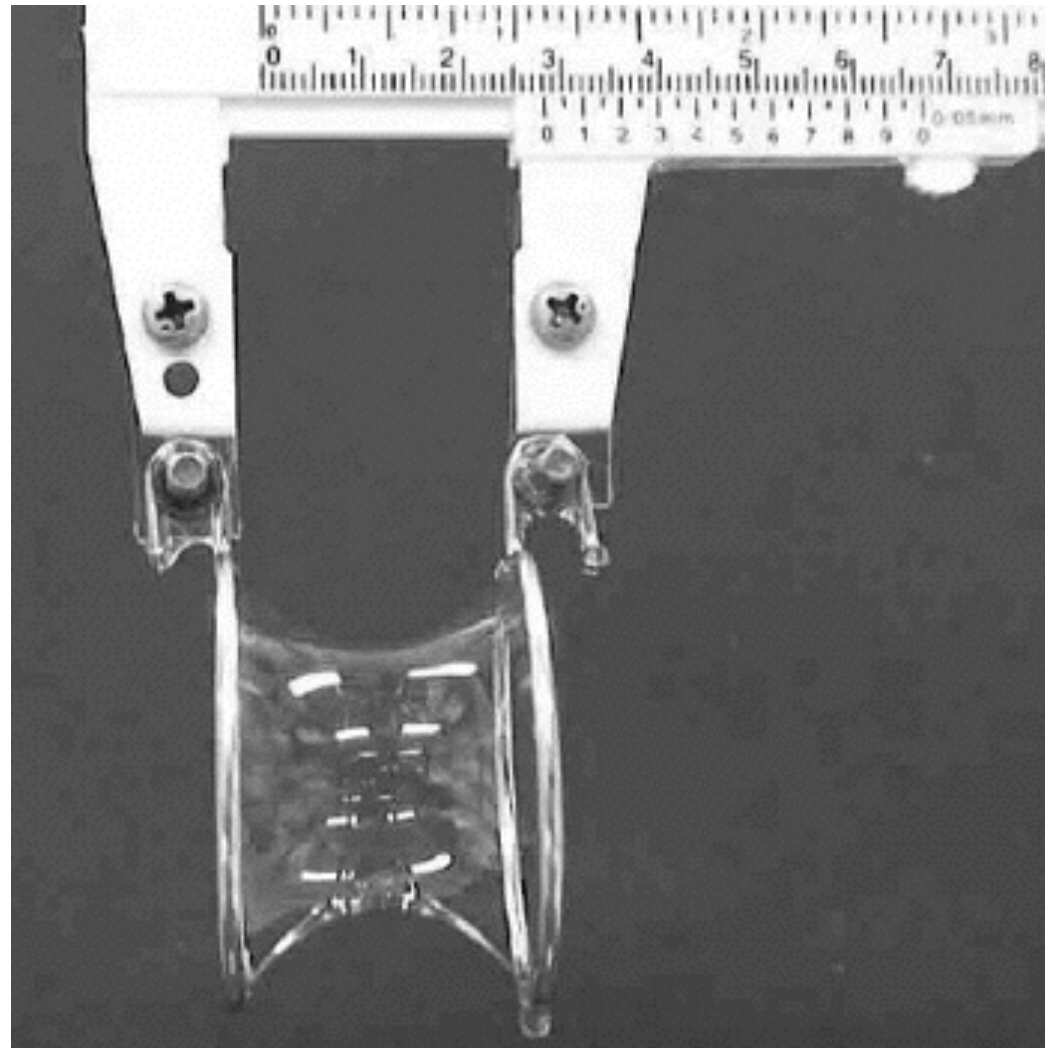# Dimensional analysis

0.025 SEC.
N

100 METERS

Elasticity

Calculus of variations

$$
\begin{aligned}
\frac{\delta I[Y]}{\delta Y} &= \lim_{\epsilon \to 0} \frac{1}{\epsilon} \{ I[f(x) + \epsilon\delta(x-y)] - I[f(x)] \} \\
&= \int_{x_1}^{x_2} \left[\frac{\partial f}{\partial Y} \delta(x-y) + \frac{\partial f}{\partial Y'} \delta'(x-y) \right] dx \\
&= \int_{x_1}^{x_2} \left[\frac{\partial f}{\partial Y} - \frac{d}{dx} \frac{\partial f}{\partial Y'} \right] \delta(x-y) dx.
\end{aligned}
$$

$$
0 = \frac{\partial f}{\partial Y} - \frac{d}{dx} \frac{\partial f}{\partial Y'}
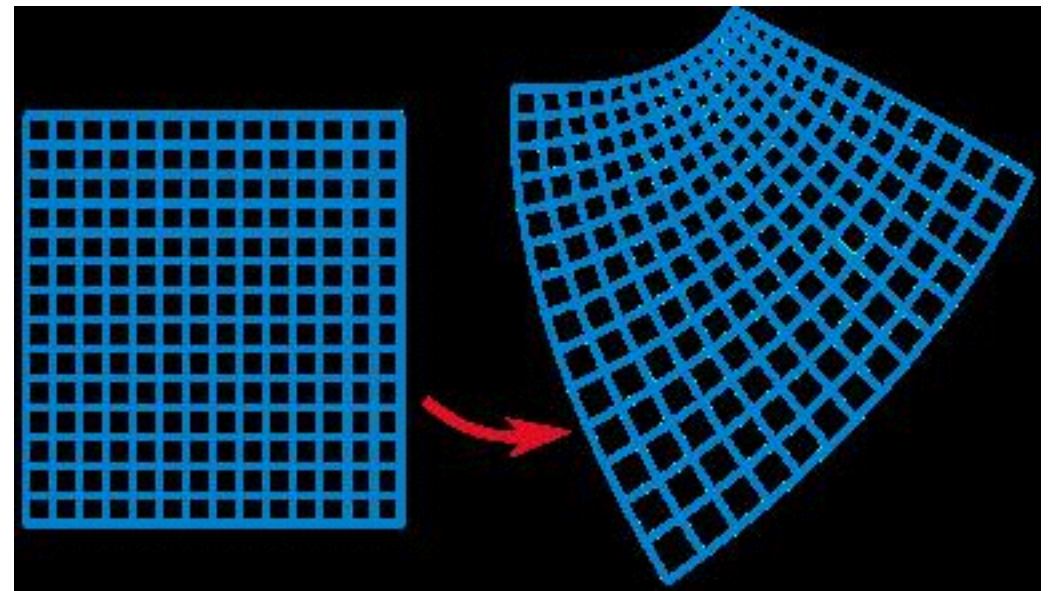$$

Surface tension

Singular perturbations

$$\epsilon \frac{d^2 u}{dx^2} + \frac{du}{dx} = 1.$$

Conformal mappings

$$\frac{dw}{dz} = \frac{\partial \phi}{\partial x} + i \frac{\partial \psi}{\partial x} = u - iv.$$
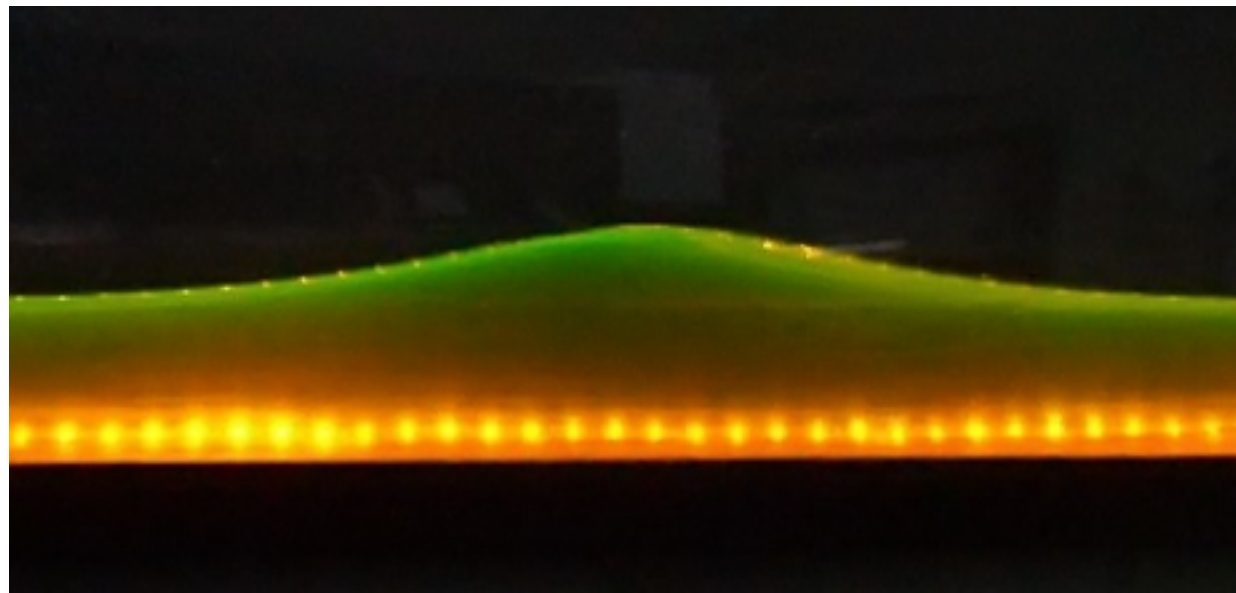
$$W(Z) = u_0 \left(Z e^{-i\alpha} + \frac{R^2}{Z} e^{i\alpha} \right) - \frac{i\Gamma}{2\pi} \ln Z.$$

Rotating flows

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \Omega \times (\Omega \times \boldsymbol{r}) = -\frac{1}{\rho} \nabla p_{\Omega} + \nu \nabla^2 \boldsymbol{u} - 2\Omega \times \boldsymbol{u},$$

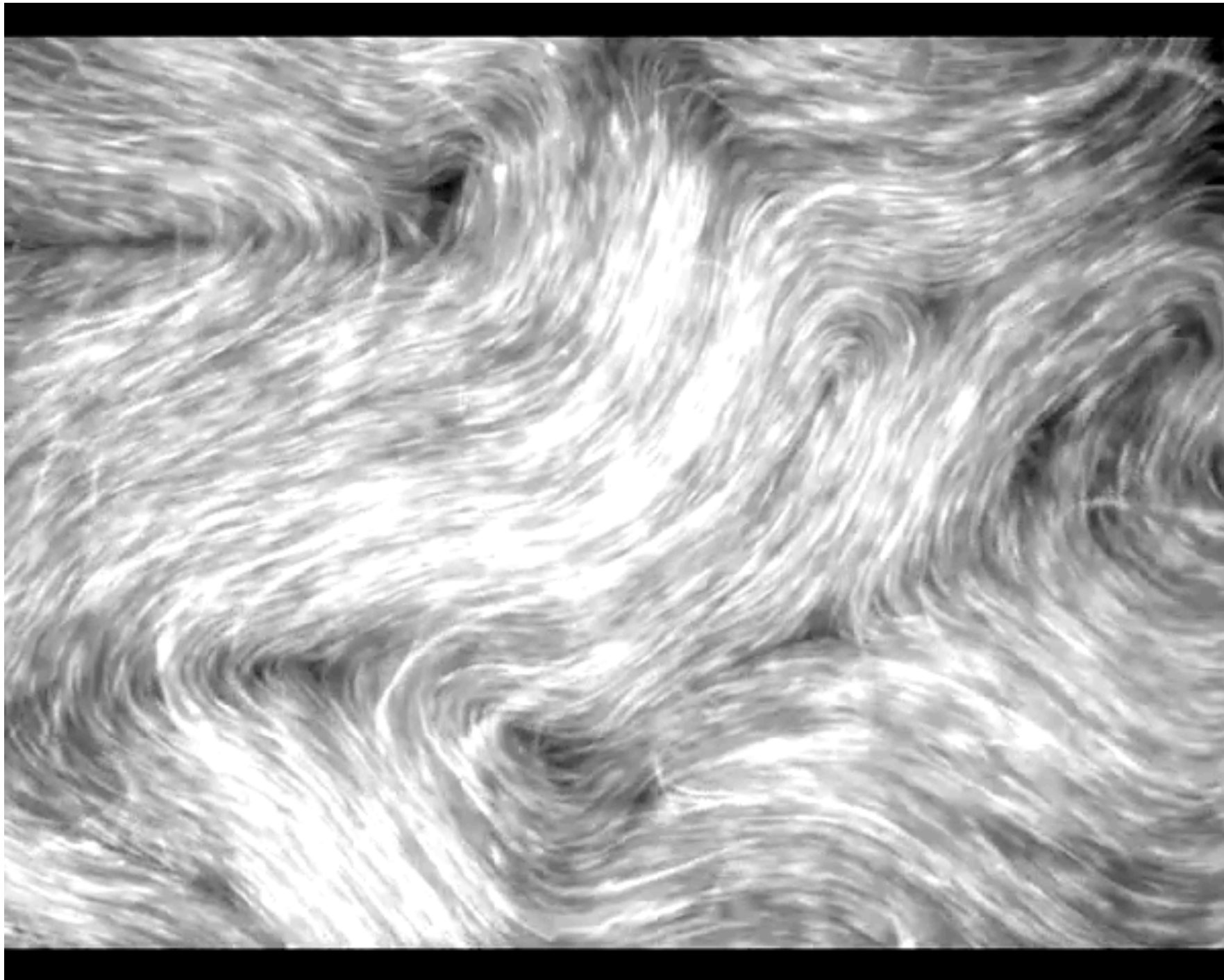$$\nabla \cdot \boldsymbol{u} = 0.$$

Taylor columns, etc

Solitons

KdV equation

Active matter

Topological defects